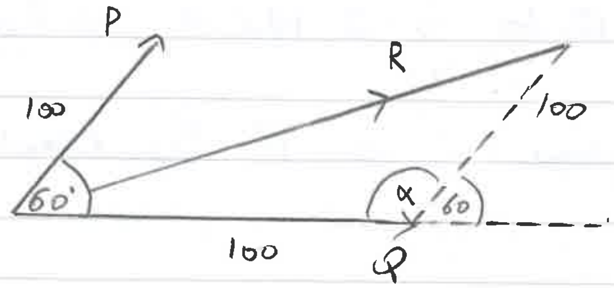
Exercise 4D : Examination Questions

(1)



$\alpha = 120$

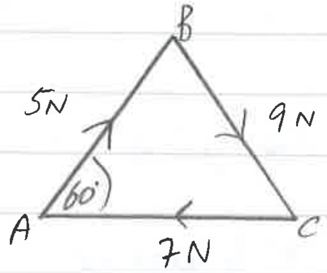
So use cos Rule.

$\therefore \; R^2 = 100^2 + 100^2 - 2(100)(100)\cos 120$

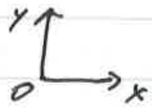
$= 30000$

$\Rightarrow R = \sqrt{30000} = 100\sqrt{3}$ N

(2)



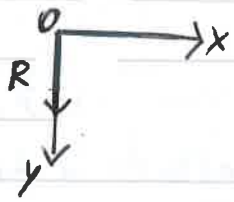
use The Frame of Reference to be



So $O_x = -7 + 5\cos 60 + 9\cos 60 = 0$ N

$O_y = 5\sin 60 - 9\sin 60 = -2\sqrt{3}$ N

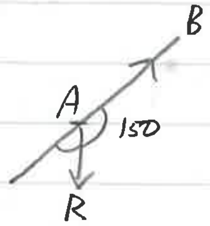
So $|R| = \sqrt{0^2 + (-2\sqrt{3})^2} = 2\sqrt{3}$ N

So R is located as: 

$;\ \theta = \tan^{-1}\left(\frac{-2\sqrt{3}}{0}\right) = 90°$

So R is at $60 + 90 = 150°$ to AB as in diagram

or at $30°$ to AB if BA is extended to lower left



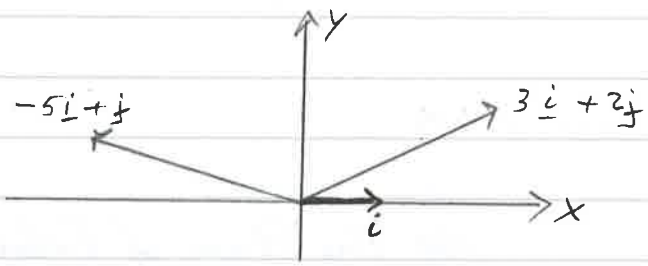
(3) $O_x = 3 - 6\cos 60 = 0$ So NO $\underline{i}$ component

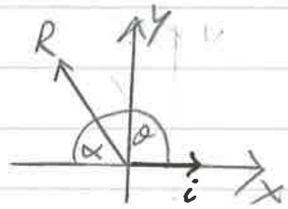
$O_y = -2 + 6\sin 60 = 3.196\ N$ = Resultant

Since The $\underline{i}$ component is 0 The Resultant is in The +ve y direction.

<u>or</u> do $\theta = \tan^{-1}\dfrac{3.196}{0} = 90^\circ \Rightarrow$ +ve y direction

(4)



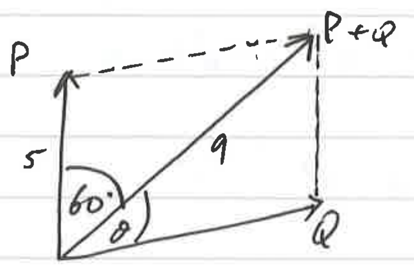


$R = 3\underline{i}+2\underline{j} - 5\underline{i}+\underline{j} = -2\underline{i}+3\underline{j}$, i.e.

$\therefore |R| = \sqrt{(-2)^2+3^2} = \sqrt{13}\ N$

and $\alpha = \tan^{-1}\left(\dfrac{3}{-2}\right) = 56.31 \Rightarrow \theta = 123.69^\circ$

(5)



So : $Q^2 = 5^2 + 9^2 - 2(5)(9)\cos 60$
$= 61$
$\Rightarrow Q = \sqrt{61}\ N$

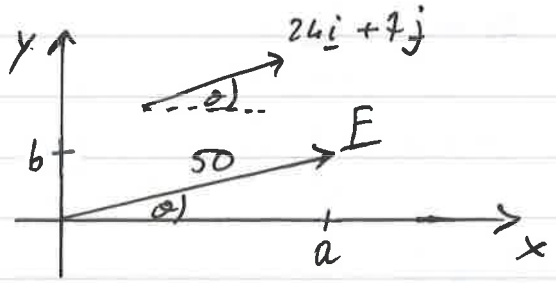
Direction in question is stated as a heading of 060°.
So answer should also be a heading, i.e. w.r.t. $\underline{P}$

So $5^2 = 9^2 + (\sqrt{61})^2 - 2(9)\sqrt{61}\cos\theta \Rightarrow \theta = 33.67$

So heading from P $= 60 + 33.67 = 093.67^\circ$

(6) See below

(7)



given $|\underline{F}| = 50$ N
& direction $= 24\underline{i} + 7\underline{j}$

Then $\quad 50 = |\underline{F}| = \sqrt{a^2 + b^2} \quad$ (1)

and

$\tan\theta = \dfrac{b}{a} = \dfrac{7}{24} \quad$ (2) $\quad$ (Same direction $\Rightarrow$ same θ to horizontal)

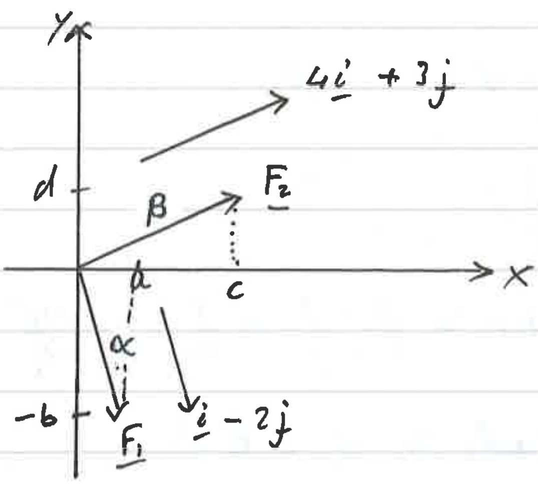
So $\quad b = \dfrac{7}{24} a$ by (2). Into (1):

$$50^2 = a^2 + \left(\frac{7}{24}a\right)^2 \Rightarrow \frac{50^2}{1^2 + \left(\frac{7}{24}\right)^2} = 2304 = a^2$$

$\therefore a = 48$, and by (1), $b = 14$

$\therefore \underline{F} = (48\underline{i} + 14\underline{j})$ N

Now



So $|\underline{F_1}| = \alpha = \sqrt{a^2 + b^2}$ (a)

& $|\underline{F_2}| = \beta = \sqrt{c^2 + d^2}$ (b)

Also $\tan\theta_1 = \dfrac{b}{a} = -\dfrac{2}{1}$ (c)

& $\tan\theta_2 = \dfrac{d}{c} = \dfrac{3}{4}$ (d)

(See comment next to (2) of Ex (7) above)

So by (c) : $b = -2a$. Into (a) : $\alpha = \sqrt{a^2 + 4a^2}$

$$= a\sqrt{5}$$

So $a = \frac{1}{\sqrt{5}}\alpha$

& $b = -\frac{2}{\sqrt{5}}\alpha$

$\therefore \underline{F_1} = \left(\frac{1}{\sqrt{5}}\alpha\right)\underline{i} - \left(\frac{2}{\sqrt{5}}\alpha\right)\underline{j}$

By (d) : $d = \frac{3}{4}c$. Into (b) : $\beta = \sqrt{c^2 + \frac{9}{16}c^2}$

$= \frac{5}{4}c \Rightarrow c = \frac{4}{5}\beta$

$\therefore d = \frac{3}{5}\beta$

$\therefore \underline{F_2} = \left(\frac{4}{5}\beta\right)\underline{i} + \left(\frac{3}{5}\beta\right)\underline{j}$

Resultant $\underline{F} = \underline{F_1} + \underline{F_2}$ So

$$48\underline{i} + 14\underline{j} = \underline{i}\left(\frac{1}{\sqrt{5}}\alpha + \frac{4}{5}\beta\right) + \underline{j}\left(-\frac{2}{\sqrt{5}}\alpha + \frac{3}{5}\beta\right)$$

$\therefore$

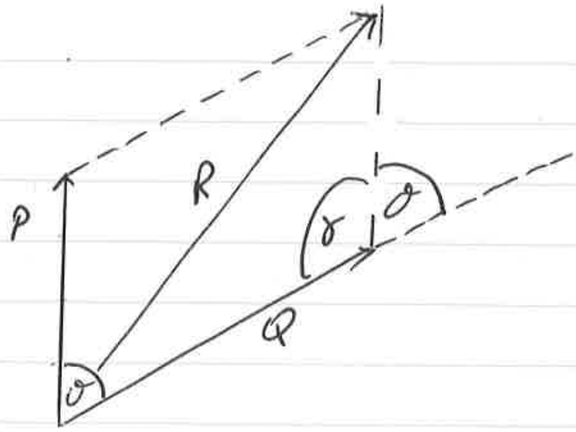
$48 = \frac{1}{\sqrt{5}}\alpha + \frac{4}{5}\beta$ (1)

& $14 = -\frac{2}{\sqrt{5}}\alpha + \frac{3}{5}\beta$ (2)

Hence $2 \times (1) + (2)$: $110 = \beta\left(\frac{8}{5} + \frac{3}{5}\right) = \frac{11}{5} \Rightarrow \beta = 50$

& $3 \times (1) - 4 \times (2)$ leads to $\alpha = 8\sqrt{5}$.

(8)



$\gamma = 180 - \theta$

Cos Rule

$R^2 = P^2 + Q^2 - 2PQ\cos(180-\theta)$

So $R^2 = (\sqrt{12})^2 + 2^2 - 2(\sqrt{12})(2)\cos(180-\theta)$

$= 12 + 4 - 4\sqrt{12}(-\cos\theta)$

$= 16 + 4\sqrt{12}\cos\theta$ (1)

& $R^2 = (\sqrt{12})^2 + 4^2 - 2(\sqrt{12})(4)\cos(180-\theta)$

$= 12 + 16 - 8\sqrt{12}(-\cos\theta)$

$= 28 + 8\sqrt{12}\cos\theta$ (2)

(1) − (2) : $0 = 16 - 28 + \cos\theta(4\sqrt{12} - 8\sqrt{12})$

So $\cos\theta = \dfrac{12}{-13.856} \Rightarrow \theta = 150^\circ$

Then from (2) : $R^2 = 28 + 8\sqrt{12}\cos 150 = 4$

$\Rightarrow R = 2N$